Technical Comments

Comment on "A Direct Matrix Method for the Divergence Problem"

William P. Rodden*
Los Angeles, Calif.

It is curiously refreshing to be reminded from time to time of the struggle that accompanies the advancing state-of-the-art of the aeronautical sciences. Lest we take our present position too much for granted, Ref. 1 has just appeared from out of the past to recall the period of the 1950's during which the field of static aeroelasticity made the transition from the slide-rule to the high-speed digital computer. The following historical recollections are offered as a more complete review of that dynamic period of progress.

Equation (4) of Ref. (1) was almost implicit in Eq. (8–55) of Bisplinghoff, Ashley, and Halfman.[2] The definition of static and oscillatory aerodynamic influence coefficients (AIC's) in Ref. 3 [Eq. (6) of Ref. 1 appears as Eq. (8) in Ref. 3] provided for a general formulation of the various aeroelastic problems which was outlined in the introduction to Ref. 4. A general purpose computer program was developed in Ref. 5 for the static aeroelastic analysis of the problems of rigid and flexible load distributions, divergence, estimation of rigid and flexible static and dynamic stability derivatives, and the correction of wind-tunnel data measured on flexible models. Both structural influence coefficients and AIC's were used and Eq. (4) of Ref. 1 is found as Eq. (23) in Ref. 5.

The example problem used to demonstrate the computer program of Ref. 5 was also the jet transport wing of Bisplinghoff, Ashley, and Halfman.[2] However, the AIC's were based on the subsonic lifting surface theory (at a Mach number of zero) of Runyan and Woolston.[6,7] The first two divergence dynamic pressures were calculated to illustrate the divergence option, and were $q_1 = 3786$ and $q_2 = 26{,}951$ psf. The fundamental divergence dynamic pressure corresponds to the velocity $V_1 = 1784$ fps at sea level, with which the value in Ref. 1 using lifting line theory agrees well.

Later calculations of the divergence characteristics of the jet transport wing were performed using incompressible strip theory in connection with a method for transient flutter analysis.[8] The five divergence dynamic pressures were found to be $q_1 = 2397$, $q_2 = 9510$, $q_3 = 23{,}555$, $q_4 = 38{,}208$, and $q_5 = 69{,}914$ psf. The fundamental divergence velocity at sea level is $V_1 = 1420$ fps, as was mentioned in Ref. 8. If the aspect ratio correction used in Refs. 1 and 2 is made, the divergence velocity at sea level becomes $V_1 = 1950$ fps and agrees with the result in Ref. 2, p. 440.

The concern in Ref. 1 over the null eigenvalues from bending is unwarranted. The five nonzero eigenvalues of the example wing (using strip theory) were found easily by the power method because of their wide separation. The sixth through tenth eigenvalues were not zero (infinite divergence speeds) because of round-off errors in the eigen-matrix deflation required in the power method; however, the sixth eigenvalue was four orders of magnitude lower than the fifth and thereby gave some clue to what should have been expected.

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* Consulting Engineer. Associate Fellow AIAA.

Reply by Author to William P. Rodden

V. T. Nagaraj*
Hindustan Aeronautics Ltd., Bangalore, India

The author wishes to thank Dr. Rodden for his comments. One of the objects of the Note was to draw attention to the procedure (which the author believes to be superior to the assumed mode method both for torsional and chordwise divergence calculations). But the main aim was to point out a method of overcoming the difficulties associated with the zero eigenvalues arising from bending.

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* Project Engineer.

Comments on "Synthesis of Tire Equations for Use in Shimmy and Other Dynamic Studies"

R. J. Black*
The Bendix Corporation, South Bend, Ind.

The study by Rogers and Brewer[1] proposes an empirical determination of the differential equations governing tire motion for aircraft shimmy with the central idea that their method is accurate and practical while other methods are either inaccurate or impractical.

In particular they cite the 5 parameter modified Moreland theory of Ref. 2 and 3 as being inaccurate because it does not agree with their tire moment vs reduced frequency curves at higher values of reduced frequency. Since the modified Moreland method has been used successfully in many cases and is more straightforward than the method of Rogers and Brewer, it is felt that some clarification of the importance of the short wavelength response of a tire is in order.

The important range of reduced frequencies for aircraft shimmy analysis is the range of $0.1 < \Omega < 2.0$ rad/ft. This

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* Analytical Mechanics Department, Energy Controls Division.

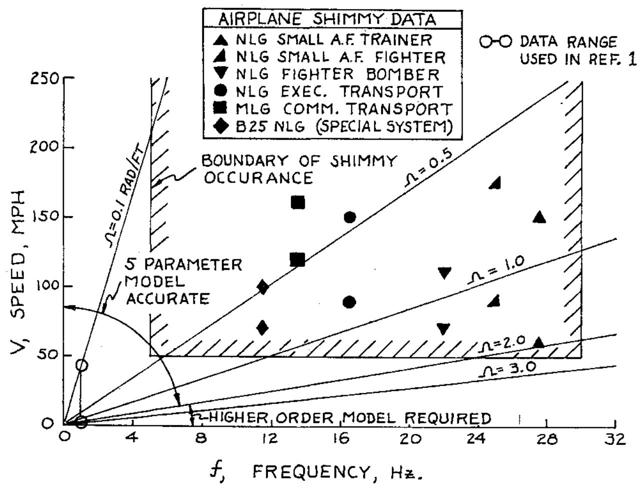

Fig. 1 Boundary of shimmy occurrence and its relationship to reduced frequency.

is because the shimmy frequency of modern aircraft main and nose landing gear systems falls in the range of 5–30 Hz while the speed range in which shimmy is encountered is almost always above 50 mph with increased severity as the speed is increased.

Figure 1 shows this boundary in a speed vs frequency plot and also shows the straight lines representing constant values of reduced frequency. Examples of shimmy of several representative aircraft are also shown. No modern aircraft examples could be found by the author which fell outside of the crosshatched boundary.

The boundary of $0.1 \leq \Omega \leq 2.0$ essentially includes the $V \geq 50$ mph, $5 < f < 30$ Hz boundary and is even less restrictive than the latter for frequencies below 24 Hz or speeds below 200 mph.

The present author has found that the 5 parameter modified Moreland model gives an accurate representation of the tire dynamics within the reduced frequency range of $0.1 \leq \Omega \leq 2.0$. Figure 2 shows RMS frequency and stability errors vs reduced frequency for two tires tested in a rolling tire test machine undergoing free vibration. The test machine is described in Ref. 2. The tests involved both wheel yaw and sideslip with variations in the proportions of each achieved by variations in the trail length of the wheel about a vertical pivot axis. The rms errors shown are the differences between the analytical and experimental dieaway ratios and frequencies, based on results for all of the trail lengths used for a particular vertical load and speed combination. Analytical results are derived using the 5 parameter tire model.

As noted on Fig. 2, the average rms error is only 6% for the amplitude ratio and 4.4% for the frequency. It certainly is not true that the analytical model "suffers from a lack of accuracy in characterizing tire behavior" as stated by Rogers and Brewer in Ref. 1, since the correctly represented vibratory motions of the tire test machine are directly determined by the tire behavior.

Considerable extrapolation is necessary to apply the data of Ref. 1 to the speed and frequency range in which shimmy is encountered. Using the assumption that velocity effects on the force and moment response are a higher order effect is highly questionable. Data taken by Pacejka[4] at 1.0, 2.16, 4 and 8 Hz over a range of speeds from 10 to 100 km/hr, indicates that the variation in tire response due to speed effects for a fixed value of reduced frequency is of the same order of magnitude as that from the reduced frequency at a fixed value of speed. His peak speed is only 62.1 mph which is well below takeoff or landing speeds of modern aircraft.

Rogers and Brewer state in Ref. 1 that the net resultant of the forces developed during wheel sideslip and yaw is a cornering force and a self-aligning moment. Actually, resolution of the footprint forces at the wheel center also

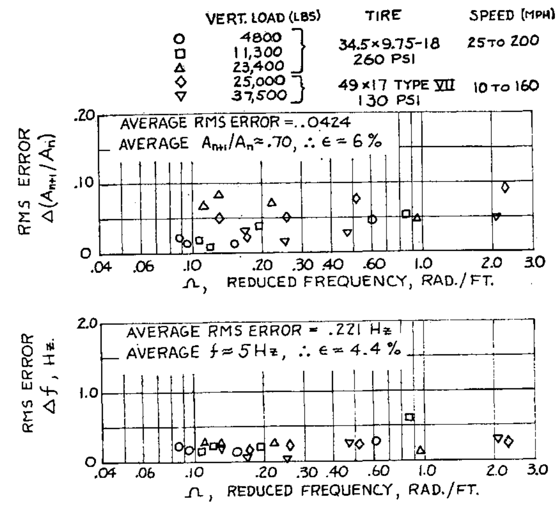

Fig. 2 Root mean square differences between analytical and experimental vibrations of a rolling tire.

requires an overturning moment even with pure yaw or pure sideslip. The overturning moment is significant and must be included in a shimmy analysis of an aircraft landing gear. If the moment caused by the vertical load acting on the displaced footprint is ignored the overturning moment can be approximated by the product of the cornering force and the distance from the wheel axis to the ground. However, the moment caused by the displaced footprint is not always small, especially for aircraft tires.

As a general comment, the stated objective of Rogers and Brewer was the development of a practical and accurate method for representation of transient tire forces. Their model in its general form contains 13 parameters.

Several more parameters would be required to represent the overturning moment. Also camber would have to be accounted for since it is important in landing gear shimmy; thus, additional parameters would have to be added. Speed effects would require that each of the parameters be determined as a function of speed. Since the parameters are very sensitive to vertical load the entire sequence of vibratory testing and data fitting would have to be repeated for the range of operating loads for the particular tire. Considering the large number (15–20) of velocity and load dependent parameters which must be established and the fact that there is no existing facility on which these parameters may be conveniently measured, I take exception to Rogers' and Brewer's claim of simplicity and practicality.

It appears that a much more practical approach would be to simply add one or two additional coefficients to the moment expression used in the five parameter model if one desires to carry its applicability into the short wavelength range of tire motions. Collins has done this in Ref. 5 and obtained very reasonable results well into the short wavelength range. His results also include wheel speed effects independent of reduced frequency, similar to those found experimentally by Pacejka.

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